Mathematical Biology Take Home Final Exam
Due Dec 6, 2011, 5pm
Submit your answers on Blackboard via the Safe Assignment link on the Assignments Page. Both instructors have access to the three courses on Blackboard.

You may use Maple, class notes, the text, or publish papers, and you may ask Dr. Wethey for help with Maple code. Other than that, you may not consult with anyone.

1. (33 pts) The Leslie-Lefkowitch matrix for a population with classes I, II, III, IV is:

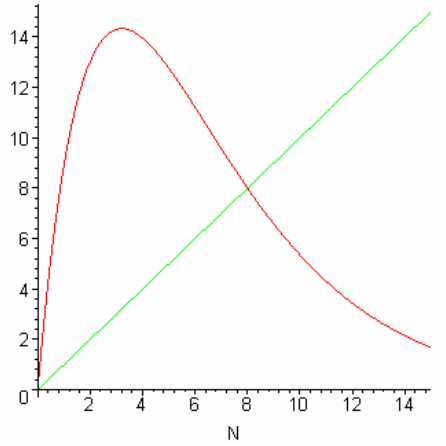
$$A := \begin{bmatrix} 0 & .4 & 1 & 0 \\ .9 & 0 & 0 & 0 \\ 0 & .8 & 0 & 0 \\ 0 & 0 & .2 & .1 \end{bmatrix}$$

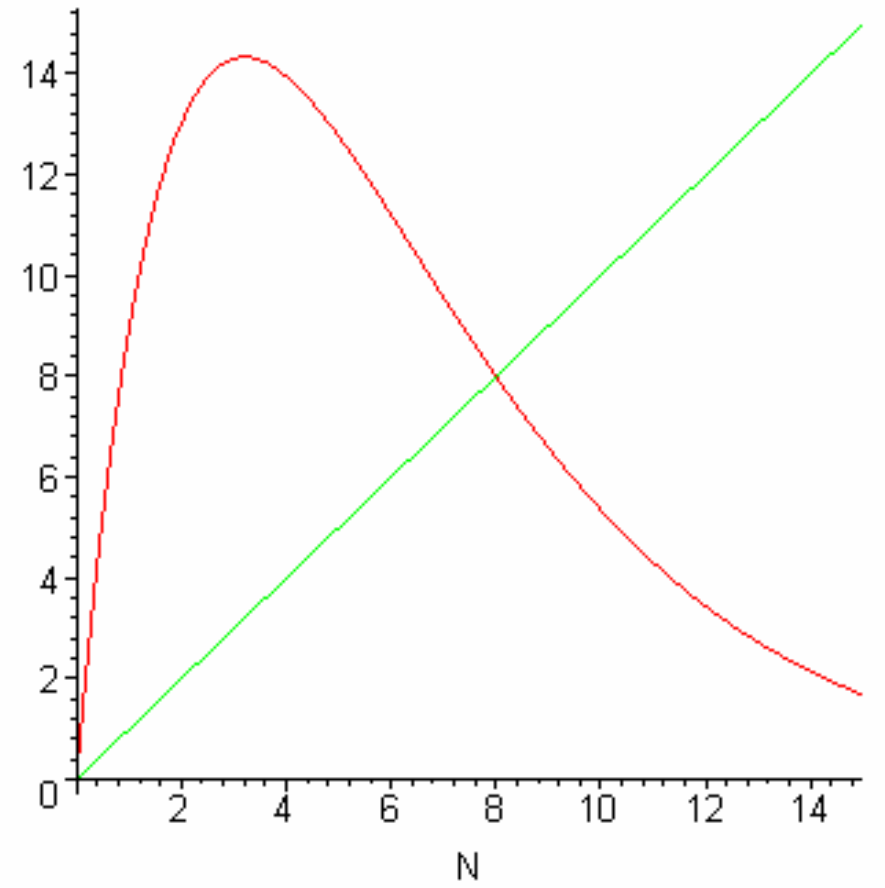
   a. Does this model describe age classes or stages? Why? (Suggestion: what is the significance of the last entry in the bottom row?) Which classes are reproductive?
   b. If the current population consists of 100 individuals in class II, how many individuals will be in each class two time steps from now?
   c. The eigenvalues of this system are

$$ev := .1000000000,\ 1.029316809,\ -.5146584047 + .6592570217\ I,\ -.5146584047 - .6592570217\ I$$

   which is the dominant eigenvalue, and does it indicate long term growth, decline, or stability?
   The eigenvector associated with the dominant eigenvalue is
   $[-.7756329972, -.6781874046, -.5270971183, -.1134375521]$.
   What is the long term age/stage distribution in percent terms?

2 (33 pts) The plot below is $N_{t+1} = F(N_t)$. A larger copy is on the next page.





a. Based on the graph, what is the non-zero steady state N*?
b. If $N_0 = 2$, use the graph to compute N_4.
c. Discuss the stability properties of the two equilibria, and the local behavior of the system near the two equilibria in terms of the appropriate tangent line. Draw the tangent line for each equilibrium, and determine its slope using the axes and a ruler.

2. (33 pts) Consider a predator-victim system described by:

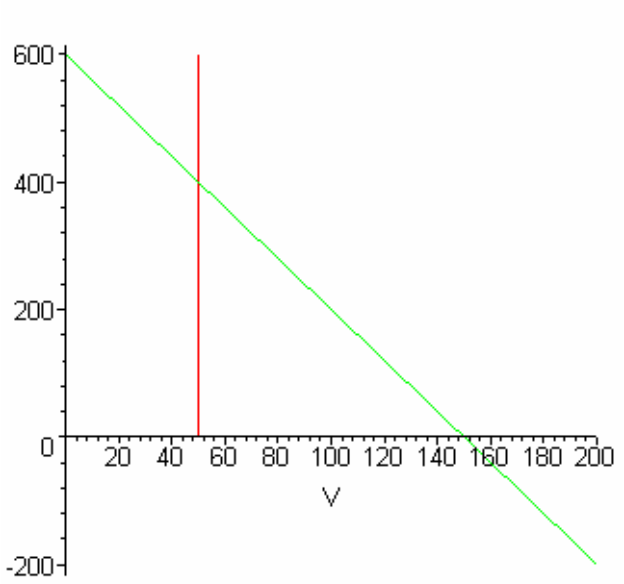
$$\frac{dV}{dt} = aV\left(1 - \frac{V}{K}\right) - bVN$$

$$\frac{dN}{dt} = cVN - dN$$

All parameters (a,b,c,d,K) are positive.

a. What happens to the victim when the predator population is zero? What happens to the predator when the victim population is zero? (do the populations increase, decrease, or remain constant)

b. The following is a phase plot of the system where a=0.6, b=0.001, c=0.008, d=0.4, K=150. Which nullcline is dN/dt=0 and which is dV/dt=0?



c. If you start a population in each of the 4 regions, what happens to N (increase, decrease, remain constant)? What happens to V in the same starting conditions? Use arrows on the plot to indicate the direction of change.

d. What is the Jacobian matrix of the system in symbolic form?

e. What numerical values does the Jacobian matrix have when evaluated at the two-species equilibrium?

f. What do the eigenvalues of the Jacobian tell you about the local stability of the system at the two-species equilibrium? Explain your reasoning.